

FRACTURE MECHANICS VALIDITY LIMITS

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SUMMARY

The consideration of fracture behavior in design is of vital concern to the aerospace industry. Fracture behavior is characteristic of a dramatic loss of strength compared to elastic plastic deformation behavior. Fracture parameters such as K , G , J , and J_M have been developed and exhibit a range within which each is valid for predicting growth. Each is limited by the assumptions made in their development: all are defined within a specific context. For example, the stress intensity parameter, K , and the crack driving force, G , are derived using an assumption of linear elasticity. To use K or G , the zone of plasticity must be small as compared to the physical dimensions of the object being loaded. This insures an elastic response, and in this context, K and G will work well. Rice's J -integral [1] has been used beyond the limits imposed on K and G . J requires an assumption of nonlinear elasticity, which is not characteristic of real material behavior, but is thought to be a reasonable approximation if unloading is kept to a minimum. As well, the constraint cannot change dramatically (typically, the crack extension is limited to ten-percent of the initial remaining ligament length). Rice, et al [2] investigated the properties required of J -type parameters, J_x , and showed that the time rate, dJ_x/dt , must not be a function of the crack extension rate, da/dt . Ernst [3] devised the modified- J parameter, J_M , that meets this criterion. J_M correlates fracture data to much higher crack growth than does J . Ultimately, a limit of the validity of J_M is anticipated, and this has been estimated to be at a crack extension of about 40-percent of the initial remaining ligament length.

None of the various parameters can be expected to describe fracture in an environment of gross plasticity, in which case the process is better described by deformation parameters, e.g., stress and strain. In the current study, various schemes to identify the onset of the plasticity-dominated behavior, i.e., the end of fracture mechanics validity, are presented. Each validity limit parameter is developed in detail, and then data is presented and the various schemes for establishing a limit of the validity are compared. The selected limiting parameter is applied to a set of fracture data showing the improvement of correlation gained.

INTRODUCTION

The safety and reliability of structures has always been a matter of vital concern to the aerospace industry. In this respect, fracture mechanics is especially useful, since it can provide a quantitative description of the capability of structural parts to tolerate flaws. The initial conditions considered for fracture mechanics were quasi-linear elastic conditions (LEFM). The methods were eventually developed further to include cases where yielding was not confined to a small region.

The parameters developed for use in the LEFM technology, G and K , are efficient as fracture predicting tools as long as the material responds in a linear-elastic manner. This occurs when the plastic

zone present at the tip of the crack is found to be much smaller than the ligament dimensions. To consider a more realistic class of problems, where the plasticity was not limited to a very small region, Elastic-Plastic Fracture Mechanics [EPFM] Methods were developed. The J-Integral was developed by Rice [1] by assuming non-linear elasticity, and was thereby limited in the range of applicability. The requirements for J-control are small crack tip displacement (CTOD), proportional loading, and small crack extension. Hutchinson and Paris [4] defined ω which evaluates the degree of nonproportionality, with the significance that some unloading can be tolerated without invalidating J, as long as the ω -criterion holds. The last requirement was established to avoid crack growth to an extent that the constraint environment controlling fracture changes. Constraint as used here is the degree of triaxiality of the stress field.

TESTING RESULTS

Tests were conducted according to ASTM E1152-87 [5] with intermediate crack lengths determined by using unloading compliance data. Crack fronts had considerable curvature and a linear averaging was used to produce a single length dimension. The curvature can affect the crack length-versus-compliance relationship and the intermediate crack lengths were adjusted using the curvature correction discussed in ASTM E647-91 [6].

Figure 1 shows collections of the J_M -resistance ($J_M R$) data for the aluminum and the nickel alloys. The $J_M R$ -curve format was selected because $J_M R$ -curves correlated data to a higher level of crack extension while the JR-curves progressed towards constant J. Two observations can be made of the resistance curves presented in these graphs: (1) the resistance data shows a broad range of behavior, and (2) three separate trends of behavior appear for the aluminum, while two emerge for the nickel.

After some degree of crack growth, many of the $J_M R$ -curves exhibit an inflection point and become concave-upwards. This is thought to be the signalling of a change of behavior from a regime controlled by fracture mechanics into one dominated by plasticity. To properly evaluate the effect of constraint in fracture requires that the data be qualified as representative of fracture mechanics behavior, such that the only variation is the constraint and not a change in the behavioral mode.

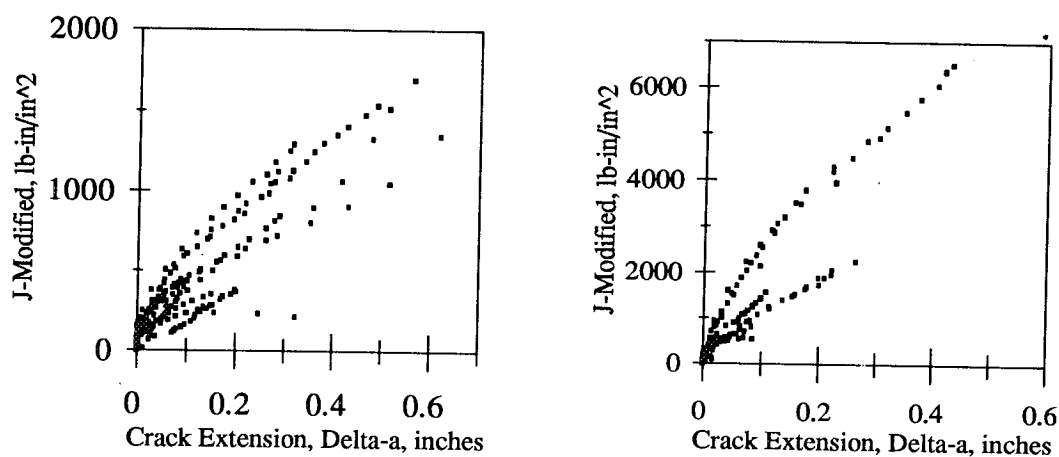


Figure 1: $J_M R$ behavior of Task I Al6061-T651 and IN718-STA1 fracture specimens: (a) $J_M R$ curves for aluminum CT specimens, (b) $J_M R$ curves for nickel CT specimens.

FRACTURE MECHANICS VALIDITY LIMITS PARAMETERS

The matter for current consideration is an assessment of the limits within which fracture mechanics parameters must operate and to ultimately apply those limits to the data set. In the process of loading, structures made of ductile materials may respond to the loading with fracture mechanics behavior, and with sufficient crack extension, the behavior will become plasticity-dominated. A method must be established to properly separate the region of fracture-dominated behavior from that of plasticity-dominated material behavior. This segregation of behavior would be necessary even if a parameter were found to describe crack growth behavior throughout the whole fracture mechanics regime: a limit to the fracture mechanics regime exists. This qualification of fracture behavior will be essential to an investigation of the three-dimensional aspects of fracture for two reasons: (1) the resulting material fracture resistance data collected for planar specimens might be applied to the general, three-dimensional case, and (2) the fracture mechanics validity limits analysis in two-dimensions might produce some insight applicable to three-dimensional fracture.

Several commonly used fracture parameters are subject to very confining limits. For example, for the stress intensity parameter, K , the plastic zone must be small with respect to relevant dimensions of the structure. This has been expressed as follows [7]:

$$\rho \equiv \frac{\pi}{8} \left(\frac{K}{\sigma_o} \right)^2 \ll B, b, a \quad (1)$$

Equation 1 gives an estimate of the plastic zone size, ρ , while σ_o is the flow stress. The parameter, ρ , must be small versus the specimen dimensions for K to be valid.

A second fracture parameter used is the J -integral [1]. The development of J requires an assumption of nonlinear elasticity to establish the path-independent nature of the integral. Nonlinear elasticity does not faithfully represent actual structural material behavior, where energy is dissipated and permanent deformation occurs. The elasticity assumption may still suffice as long as unloading is avoided. The first limit of J , required to assure the assumptions, is that the crack tip opening displacement (CTOD) must be small compared to ligament dimensions. This is necessary to ensure small deformation theory and is expressed in a ratio with the ligament length:

$$\rho_J \approx \frac{b_o}{CTOD} \equiv \frac{b_o \sigma_o}{J} \gg 1 \quad (2)$$

A minimum value of $\rho_J \geq 20$ to 25 is generally accepted to be the limit of J -controlled behavior. Other limits of J exist. Though crack growth causes unloading and pronounces the permanent deformation behavior, the inaccuracy due to the deviation from nonlinear elasticity can be kept to an acceptable level if certain limits are held. For J , the additional validity limits are expressed [4,8,9] as:

$$\omega = \frac{b}{D} = \frac{b}{J_{lc}} \frac{dJ}{da} \gg 1 \quad (3)$$

$$\Delta a \leq 0.1 \times b_o \quad (4)$$

In equation 3, D defines the area associated with nonproportional loading. Equation 4 limits the crack growth to small enough that the constraint does not change. As long as the limits expressed in equations 3 and 4 are obeyed, J is considered valid for predicting fracture behavior.

Ernst [10,11] developed the modified J -integral, or J_M , that relaxed the tight limits imposed on J . The J -integral was developed assuming nonlinear elasticity, i.e., the deformation process was considered to be reversible. This assumption was acceptable within the limits of ω . With J , the load versus displacement (P - v) and crack length-versus-displacement (a - v) records were assumed to be path-independent.

Rice, Drugan, and Sham [2] determined that in the presence of a growing crack, any J -type parameter, say J_X , must have a rate, dJ_X/dt , that is independent of the crack growth rate, da/dt . Ernst [10,11] introduced the modified J -integral (J_M) which complies with this requirement:

$$J_M = G + \int_0^{v_{cp}} \left. \frac{\partial J_{pl}}{\partial v_{cp}} \right|_a dv_{cp} \quad (5)$$

J_M assumes real plasticity, and follows the actual, irreversible process, with the change in plastic displacement always greater than or equal to zero, i.e., $dv_{cp} \geq 0$ [11]. Whereas J_{pl} is the area between two "calibration" (i.e., non-growing crack) curves (load-versus-plastic displacement) of like specimens of infinitesimally different crack length, $J_{M,pl}$ was defined as the change in area between the load-versus-plastic displacement curves of two specimens with growing cracks where an infinitesimal difference in the crack length is always maintained. J_M meets the Rice-criterion, and includes some of the irreversibility of the fracture event. This allows fracture characterization to a much greater extent of crack growth. To compare J and J_M a typical J -resistance curve, or JR curve, has been enclosed (figure 2a). This graph plots the crack extension on the abscissa versus J and J_M on the ordinate. The "ASTM box" [5] has been drawn showing the limits of J that appear in equations 2 and 4, using a value of $p_j = 20$ to predict a value of J_{max} . Note that the location where the J limit is reached, J and J_M begin to diverge noticeably. This behavior is common to all data produced in this investigation.

Limits for J_M were estimated to be a crack extension of 40-percent of the original remaining ligament for the different specimen sizes and configurations tested [11]. Later works by Ernst and Pollitz [12,13] and Ernst [14,15] have further considered the limit of validity of J_M and have suggested it to be the inflection point of the J_MR -curve.

It has been observed that with sufficient crack growth, the J_MR -curve will pass from a concave-downwards shape to one that is concave-upwards. The upwards inflection is thought to be due to a change in the deformation character. Specifically, the specimen has passed into a regime where deformation and crack growth are better described using stress-strain relationships and considering the full-field problem instead of local fracture mechanics parameters. The point at which fracture mechanics methods are no longer effective in describing the fracture event is termed herein as the *fracture mechanics validity limit* (symbolized VL). Because of this change of dominant behavior, the two regimes should be separated for evaluation. This is not a new concept. Ernst and Pollitz [12] discussed ways of extrapolating J_M in order to allow estimation of the behavior of large, thick structures from the behavior of small test specimens. The extrapolation makes use of an apparent improvement of correlation derived from using J_M instead of the J -integral parameter. They considered two options for establishing the limit of the J_M data to be used in the extrapolation: (1) the inflection point of the J_MR curve or (2) the inflection point of a plot of the plastic displacement-versus-crack extension. An assortment of schemes were then used to extend the truncated resistance curve to estimate much greater crack extension.

The inflection point observed in the J_M -R-curve is often subtle and can be difficult to determine. This suggests that other parameters might be devised to better identify the validity limit of fracture mechanics. Six candidates were considered in this investigation, although only the ones showing the most promise will be discussed in detail. Along with these, the location on the JR-curves corresponding to the maximum load during the test was determined and certain comments will be made at an appropriate time regarding the observations.

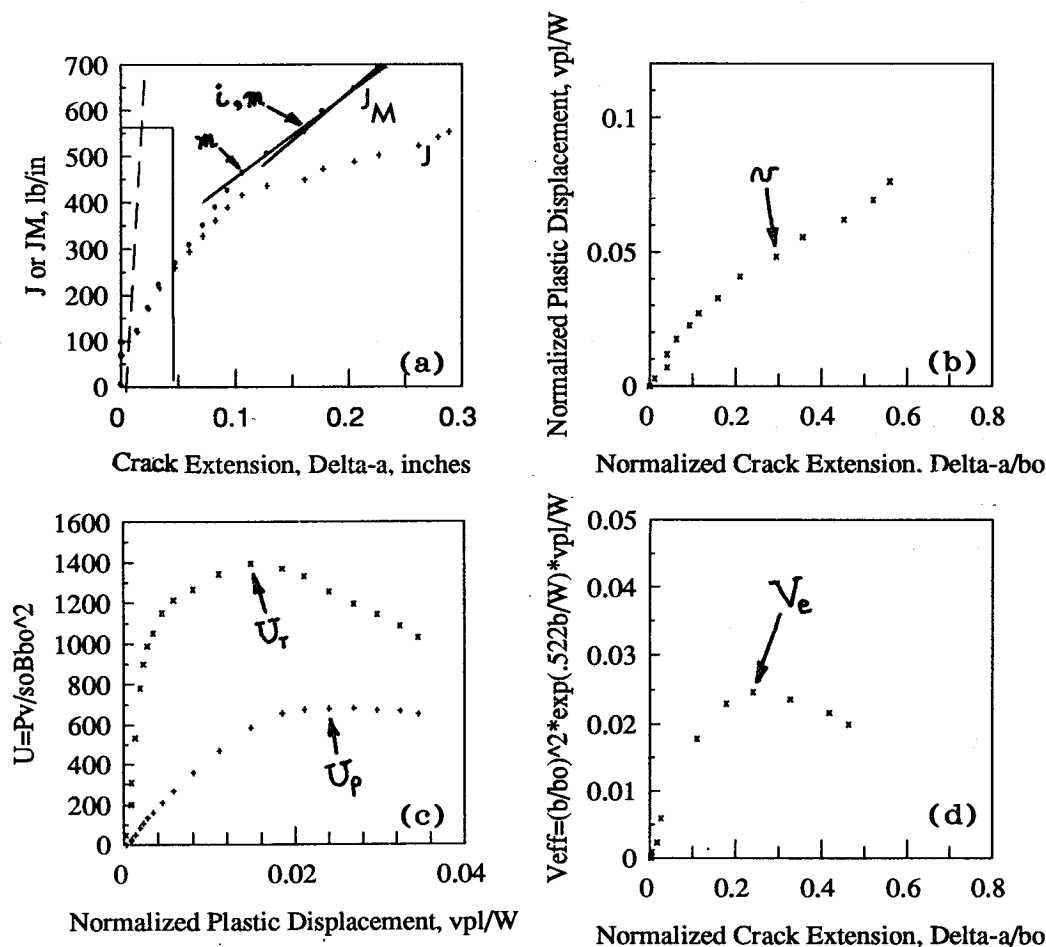


Figure 2: Various validity limit parameter definitions: (a) typical J_M -R curve showing inflection point and constant slope points, (b) inflection point of plastic-displacement-versus-crack extension, (c) product of load times plastic or total displacement, and (d) effective volume.

The first candidate VL parameter is the inflection point of the J_M -R-curve. This has been discussed as an indication in the R-curve of the onset of a new behavior, and the location is labelled "i", as in the inflection point of the J_M -R-curve, in figure 2a. One parameter which has been discarded as a candidate is the point at which the J_M -R-curve becomes straight (symbol "m"). Two such possibilities for the initiation point of constant slope are indicated in the figure. The first possibility is thought to be due to the approach of the inflection point. The second possibility is synonymous with the inflection point, i. Earlier work by Ernst [15] suggests that if fracture parameters can be scaled in the same way as plastic deformation processes, then the behavior has become plasticity-dominated, and fracture mechanics treatments are invalid. Ernst further showed that this case would be manifested as a straight line in the JR-curve format. A third candidate is the inflection point of the curve of plastic displacement-versus-crack extension (symbol "v" in figure 2b). Again, the phenomenological change is thought to be a transition from fracture-driven processes to plasticity. As with the i-criterion, this point is often subtle and not easily established. The

fourth and fifth candidates (symbolized by "U_p" and "U_t", respectively) were discussed by Ernst [15] and are the product of load-times-plastic displacement or load-times-total displacement (figure 2c). The last candidate for a fracture mechanics regime limit is the onset of constant effective volume (symbol "V_e", figure 2d). Constant volume deformation is commonly assumed for plastic deformation, and the onset of plasticity would be expected to exhibit this behavior.

The following sub-sections are devoted to the development of any formulae and theory for use in the evaluation of the various candidate VL parameters and discussions of the relevance of each.

INFLECTION POINT OF J_MR CURVE, SYMBOL "i"

The inflection of the J_MR-curve has been discussed earlier (figure 2a) as an indication of the onset of a new behavior mode. This is thought to be the result of the saturation of the ligament. This is documented by Rice, Drugan, and Sham [2], and the slope of the J_MR-curve is given by the following equation:

$$\frac{dJ_{(x)}}{da} = A_o + R_o \ln \left(\frac{R}{R_o} \right) \quad (6)$$

$$R_o = R(A_o) \quad (7)$$

In this discussion, J_(x) is J-modified, and R estimates the plastic zone size. The small scale yielding (SSY) behavior is given by

$$R = R_{ssy} \approx .25 \times E / \sigma_o^2 \quad (8)$$

In equation 8, E is Young's modulus. The behavior for large scale yielding (LSY) is a linear function of the ligament length:

$$R \Rightarrow R_{LSY} = \alpha b \quad (9)$$

When R increases to a point that $R \approx R_o$, then the J_MR curve behavior begins to change. When the slope is characterized by equation 9, the inflection point has been reached. In the process of fracture, the ligament length b will be decreased, and this would produce an increasing slope after the point of saturation.

POINT OF CONSTANT J_MR CURVE SLOPE, SYMBOL "m"

The proper scaling associated with fracture mechanics is thought to be $\Delta a_1 = \Delta a_2$ at $J_1 = J_2$. Ernst [15] has discussed the circumstances where the fracture behavior of two different specimens (proportionally sized but with different absolute dimensions) can be scaled otherwise. For example, if the behavior has become proportional crack growth (PCG), then is characterized by

$$\left(\frac{J}{W}\right)_1 = \left(\frac{J}{W}\right)_2 \quad \text{at} \quad \left(\frac{\Delta a}{W}\right)_1 = \left(\frac{\Delta a}{W}\right)_2 \quad (10)$$

This behavior is represented by a constant slope of the JR curve (figure 2a) and is not scaled properly for fracture mechanics. The current validity limit analysis has considered the constant slope of $J_M R$. Some evidence was seen in the data of the presence of a constant $J_M R$ curve slope. As was seen in figure 2a, a linear region seemed to occur either before or after the inflection point. The earlier indication may be due to the approach of the inflection point.

INFLECTION POINT OF PLASTIC DISPLACEMENT-VS.-CRACK EXTENSION, SYMBOL "v"

In the course of fracture tests, the load, total displacement, and the unloading compliance are recorded (P , v , and C , respectively). From these, and the assumption that the displacement can be decomposed into linear (v_{el}) and nonlinear (v_{pl}) parts, the nonlinear displacement can be calculated, as follows:

$$v = v_{el} + v_{pl} \quad (11)$$

$$v_{el} = P \cdot C \quad (12)$$

$$v_{pl} = v - v_{el} = v - P \cdot C \quad (13)$$

Graphs of nondimensionalized plastic displacement versus crack extension were produced (figure 2b). The use of the inflection point of the v_{pl} -versus- Δa curve as a VL parameter was discussed by Ernst and Pollitz [15]. They used a point where v_{pl}/W had grown to five-percent above the amount inferred by linearly extrapolating from the inflection point using the slope at the inflection point.

MAXIMUM OF THE PRODUCT OF LOAD TIMES DISPLACEMENT, SYMBOLS " U_P " AND " U_T "

In general, an increment of external work comes from crack growth plus a change in strain energy:

$$Pdv = Jda + \Delta U \quad (14)$$

The terms in equation 14 can be decomposed into elastic and plastic components, and the plastic portion of the expression will be considered further. This plastic portion is written from equation 14 as:

$$Pdv_{pl} = dU_{nl} + BJ_{pl}da, \quad \text{with} \quad dU_{nl} \geq 0 \quad (15)$$

If $dU_{nl} = 0$, the end of validity has been reached. This can be developed further to give a simple expression for U_{nl} by using a Ramberg-Osgood constitutive form:

$$J_{pl} = \frac{\eta}{bB} U_{nl} \Rightarrow U_{nl} = \frac{J_{pl} bB}{\eta} \quad (16)$$

$$U_{nl} = \int P dv_{pl} \propto \int v_{pl}^N dv_{pl} = \frac{v_{pl}^{N+1}}{N+1} \quad (17)$$

$$U_{nl} \propto \frac{P v_{pl}}{N+1} \quad (18)$$

Equation 17 comes by using $P \propto v_{pl}^N$. Looking at equations 15 and 18, a peak in load times plastic displacement is expected to signal a limit to fracture mechanics validity., i.e.,

$$\text{as } dU_{nl} \Rightarrow 0, \quad P \times v_{pl} \Rightarrow \text{constant} \quad (19)$$

As an alternative, the use of U_{tot} was substituted for U_{nl} . This was evaluated similarly:

$$\text{as } dU_{tot} \Rightarrow 0, \quad P \times v \Rightarrow \text{constant} \quad (20)$$

Since the theory was developed based on plasticity concepts, the use of U_{tot} may be debatable, but it has been considered, and some results discussed. The criterion shown in equation 19 is symbolized " U_p " while that in equation 20 is symbolized as " U_t ". The curves in figure 2c have been normalized to produce a stress-like quantity.

CONSTANT EFFECTIVE VOLUME, V_e

Plastic deformation is often assumed to occur at constant volume. This behavior might be applicable, given an appropriate volume, to indicate when the fracture processes have evolved to the point where fracture mechanics is no longer valid: if the change of volume goes to zero, then the fracture mechanics validity limit has been reached. An effective volume must be determined, and using the work functions or η -factors, an effective area can be defined, noting first that the η -factor arises from the assumption that the load can be separated into one function of crack length and one of plastic displacement:

$$P = g(a) \times F(v_{pl}) \quad (21)$$

$$\eta \equiv \frac{\partial P}{\partial b} \frac{b}{P} \quad (22)$$

In the case of CT specimens,

$$\eta = 2 + 0.522 \frac{b}{W} = \frac{\partial P}{\partial b} \frac{b}{P} \quad (23)$$

$$\frac{\partial P}{P} = \left(2 + 0.522 \frac{b}{W} \right) \frac{\partial b}{b} = \left(\frac{2}{b} + \frac{0.522}{W} \right) \partial b \quad (24)$$

Integrating the last equation produces a logarithmic form which can be rewritten as

$$P = C \times b^2 \exp\left(0.522 \frac{b}{W}\right) \quad (25)$$

Here, C is a constant of the integration with respect to b and can be a function of plastic displacement. Thus, C contains $F(v_{pl})$. By using the definition of stress, an effective area can be suggested:

$$A_{eff} = \frac{P}{\sigma} = \frac{C}{\sigma} \times b^2 \exp\left(0.522 \frac{b}{W}\right) \quad (26)$$

The quotient C/σ is associated with plasticity and the remaining part of equation 26 will provide the sole contribution to any changes in the effective area. From this, an effective volume for a CT specimen is written:

$$V_{eff,CT} = l_{eff} \times A_{eff} = v_{pl} \times b^2 \exp\left(0.522 \frac{b}{W}\right) \quad (27)$$

Since the effective area is written in association with the plastic portion of the load-line displacement, the appropriate length is assumed to be the plastic displacement. The effective volume criterion has been labelled " V_e " in figure 2d.

For a center-cracked tension specimen, the limit load is assumed to be the net cross-sectional area times the flow stress. This gives rise to a formula for the effective volume of a CCT specimen:

$$V_{eff,CCT} = v_{pl} \times Bb \quad (28)$$

RESULTS

If no clear evaluation of the VL parameter was obvious, then the questionable data were not included. Because of this the methods using inflection points, i.e., candidates i and v , provided fewer data. Two point and five point, unequal spacing formulae were used to estimate the derivative, dJ_M/da and $dv_{pl}/d\Delta a$, but the resulting plots possessed too much scatter and did not indicate the location of the inflection points as clearly as careful visual inspection.

Three methods, U_p , U_v , and V_e , were easily evaluated, since the maxima of the relatively smooth curves were obvious. Two means of comparing results were available: (1) crack extension at VL, and (2) J-modified at VL. In both cases, the values were normalized:

$$J_M: \Rightarrow \rho \equiv \frac{b_o \sigma_o}{J_M} \quad (29)$$

$$\Delta a: \Rightarrow \frac{\Delta a}{b_o} \quad (30)$$

The first expression can be associated with the crack opening displacement. The second equation is the fraction of the initial ligament cracked to reach the limit. Both are similar to criteria used to define the J-integral validity limits. The results were more consistent with the $\Delta a/b_o$ -form, and the ρ -form results have been excluded, because they were redundant with those of the $\Delta a/b_o$ results. Certain trends were observed in either case. These trends differed somewhat between the two materials, AL6061-T651 and IN718-STA1. It is suggested that these differences were due to the different hardening characteristics of the two materials. The nickel exhibits some hardening behavior, while the aluminum acts rather like an elastic perfectly-plastic material, exhibiting little hardening.

LOCATION OF MAXIMUM LOAD ON JR- AND J_M R-CURVES

The datum at which the maximum load was reached in each test was recorded and this location appears to be associated with the imminent divergence of J and J_M , at a crack extension of approximately five-percent of the ligament length for both materials ($\Delta a/b_o = .05 \pm .018$).

DEPENDENCE OF VARIOUS CANDIDATES ON LIGAMENT DIMENSIONS

The values of $\Delta a/b_o$ associated with each of the parameters were plotted against ligament dimensions, B, b_o and B/b_o , and the following discussion presents these results. As new parameters are brought into the discussion, they are also compared to those introduced earlier.

For the aluminum and the nickel, the inflection point of the J_M R curve is approximately constant in the $\Delta a/b_o$ form. This is especially good for $b_o > 0.4$. It should be mentioned here that, though no dependence of the i-criterion upon ligament length is stated, $\Delta a/b_o$ implicitly includes a functionality with respect to b_o . Specimen #E0 (aluminum, CT, W = 1-inch, B = 1/2-inch, $a/W = 0.75$) has been shown as an exception to the well-defined trend. An alternate formulation for the aluminum i-criterion results arises and includes specimen #E0, suggesting that a linear relationship exists for $\Delta a/b_o$ -versus- B/b_o . This would provide a " $\Delta a/B = \text{constant}$ " functionality of the i-criterion for aluminum. Specimen #E0 is different than other CT specimens, being the only one with a ligament proportion ratio, B/b_o , greater than unity. Compact tension and center cracked tension specimens produce a different constant. No data were available to suggest the value for nickel CCT specimens. The results for the nickel appear constant. If the results from specimen #E0 are omitted, the suggested values for the i-criterion validity limits are:

$$\text{CT Specimens: } \left(\frac{\Delta a}{b_o} \right)_{AL,CT}^i \approx .35, \text{ and } \left(\frac{\Delta a}{b_o} \right)_{IN,CT}^i \approx .34 \quad (31)$$

$$\text{CCT Specimens: } \left(\frac{\Delta a}{b_o} \right)_{\text{AL,CCT}}^i \approx .11 \quad (32)$$

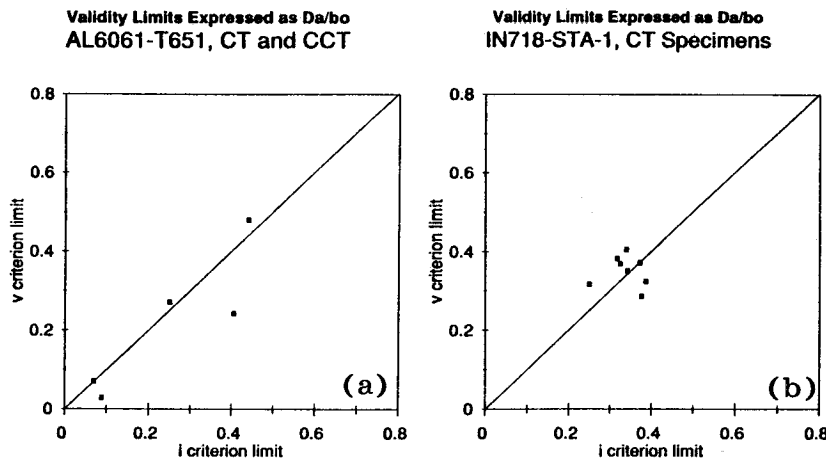


Figure 3: Comparison of inflection points of $J_M R$ curves and plastic displacement-versus-crack extension for (a) aluminum alloy and (b) nickel alloy.

The next parameter is the v -criterion, the inflection point of v_{pl} -versus- Δa . Parameter v appears to be constant. However, the results in the aluminum suggested that v may be a function of ligament length, b_o . Judging from a comparison of i and v in figure 3, i and v seem to give a one-to-one correspondence, and for the IN718-STA1, the results have a smaller range. The difference in range is suggested to be due to the different material properties and perhaps the selected range of the ligament dimensions in the nickel test matrix. The equivalence of i and v does seem reasonable.

The third and fourth criteria are U_p and U_t . U_p is not related to ligament length, b_o , but is a function of ligament thickness, B . Considering this functionality, and looking at figures 4a and 4b, no correspondence is obvious between U_p and i . A comparison between U_p and U_t appears in figures 4c and 4d. For AL6061-T651, U_t precedes U_p uniformly, and for IN718-STA1, U_t appears to be constant. Again, the lack of comparable results between the two materials must arise from differences in the material properties. No further conclusion will be drawn here, nor is one thought necessary, since U_t was included somewhat arbitrarily, as discussed earlier.

The last criterion is constant volume. V_e appears to be relatively constant for both aluminum and for nickel throughout the range of ligament dimensions and proportions investigated. None of the CCT specimens were tested to a point where a constant volume point was exhibited. For both materials,

$$\text{CT Specimens: } \left(\frac{\Delta a}{b_o} \right)_{\text{CT}}^{V_e} \approx .30 \quad (33)$$

Figures 5a and 5b show a comparison of V_e and i . For both materials, V_e and i seemed similar, although i ranges more widely for AL6061-T651. Two specimens, #E0 and #C8, possessing a ligament ratio of $B/b_o = 2$ are the notable exceptions to this. The exceptions associated with these two might be some early loss

of in-plane constraint not present with the other specimens or more simply due to ambiguity in determining i. Consideration of the Δa -shifting, mentioned earlier, will not substantially affect these findings.

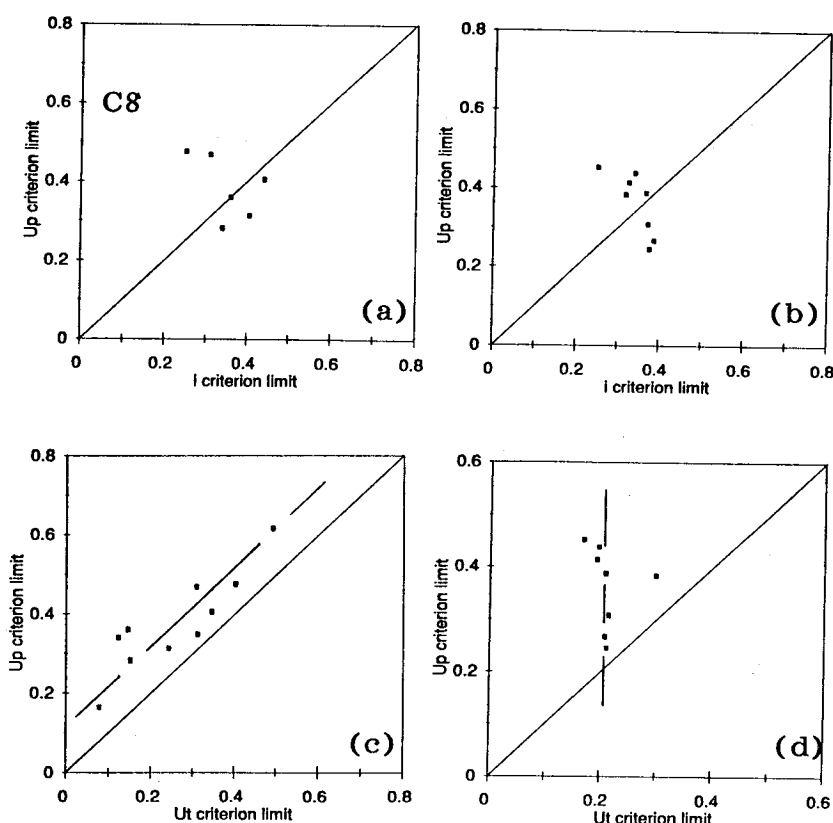


Figure 4: Comparison of load times plastic displacement criterion with inflection point of $J_M R$ curve for (a) aluminum alloy, and (b) nickel alloy, and comparison of load times plastic displacement with load times total displacement for (c) aluminum alloy, and (d) nickel alloy.

Looking at figures 5c and 5d, U_p and V_e are compared. These two criteria are very similar, but V_e precedes U_p by a significant amount. In figure 5c, one datum falls below the $U_p = V_e$ line (specimen #D6, aluminum, CCT, 2W = 2-inches, B = 1/2-inch, $a/W = 0.5$).

DISCUSSION

The first observation to be made regarding all of the candidate validity limit parameters is that those parameters that pass through a maximum are the easiest to evaluate. These include U_p , U_t , V_e . The parameters that exhibit an inflection point are much harder to evaluate with confidence, and by taking derivatives that might provide an alternative formulation with a maximum, it was found that the experimental error was exaggerated to the point where the parameter was, again, difficult to evaluate with confidence. These parameters included i and v .

The effective volume, V_e , and the inflection point of the $J_M R$ curve, i , seem to be associated primarily with the ligament length. These criteria probably represent the saturation of the ligament: when V_e and i occur, the plasticity has grown enough to reach the back face of the specimen. The problems associated

with specimen #E0 seem to conflict with this conclusion; however, the inflection point of the $J_M R$ curve for specimen #E0 was difficult to determine positively. A second possible inflection point, in the vicinity of V_e , was identifiable in the figure. The lowest value was used in the analysis. The second, higher, value of $\Delta a/b_o$ was much better, but was still low compared to the other data.

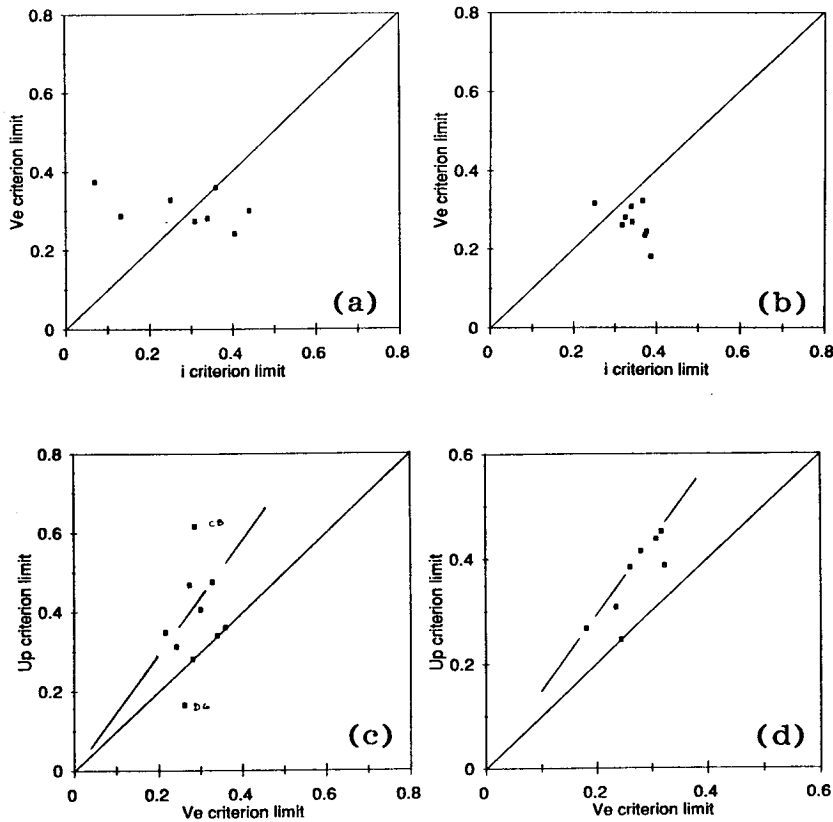


Figure 5: Comparison of effective volume criterion with inflection point of $J_M R$ curve for (a) aluminum alloy and (b) nickel alloy, and with load-times-plastic displacement for (c) aluminum alloy and (d) nickel alloy.

The observed maximum of the product of load-times-total displacement is a precursor to load-times-plastic displacement, and U_t is a conservative estimate of U_p . U_p appears to arise from both b_o and B . The functional form of U_p is not clear from the data presented here, although one suggestion is that U_p signals some overall loss of constraint. Since the crack front shape is expected to develop up to some stable shape, U_p may represent the onset of that stable crack front shape for these low-hardening materials. V_e signals a loss of in-plane constraint, only.

The lack of conclusive data for the CCT specimens is unfortunate. Where available for the aluminum CCT specimens, the results *are* considered to be consistent with the discussion, above.

Looking at i , the inflection point of the $J_M R$ curve is manifested by almost all specimens in the matrix, and although the CCT specimens do not exhibit V_e , the loss of in-plane constraint is still evident in the $J_M R$ inflection. The difficulty of establishing i in all cases is a weakness, but the earlier development and discussion suggests that the inflection point of the $J_M R$ curve is an appropriate limit to use when employing fracture mechanics methodologies. It is suggested that for compact tension specimens, subjected to tensile loading plus a bending component, V_e be used to infer i . It appears to be equivalent when B/b_o is no larger than unity, and it is much easier to evaluate.

As a result of these findings, the $J_M R$ curves were qualified by limiting the curve to that portion preceding the upwards inflection point. For a given bending-to-tension characteristic associated with a specific specimen configuration, this might be simplified to a maximum crack extension which is some percentage of the initial length of the ligament. This is expected to vary for materials of different hardening characteristics, but for the two materials used in this evaluation, the crack extension associated with V_e or i is, as follows:

$$\left(\frac{\Delta a}{b_o} \right)_{CT} \approx .30 \quad (34)$$

$$\left(\frac{\Delta a}{b_o} \right)_{CCT} \approx .10 \quad (35)$$

The $J_M R$ curves have been qualified, and the results appear in figures 6a and 6b.

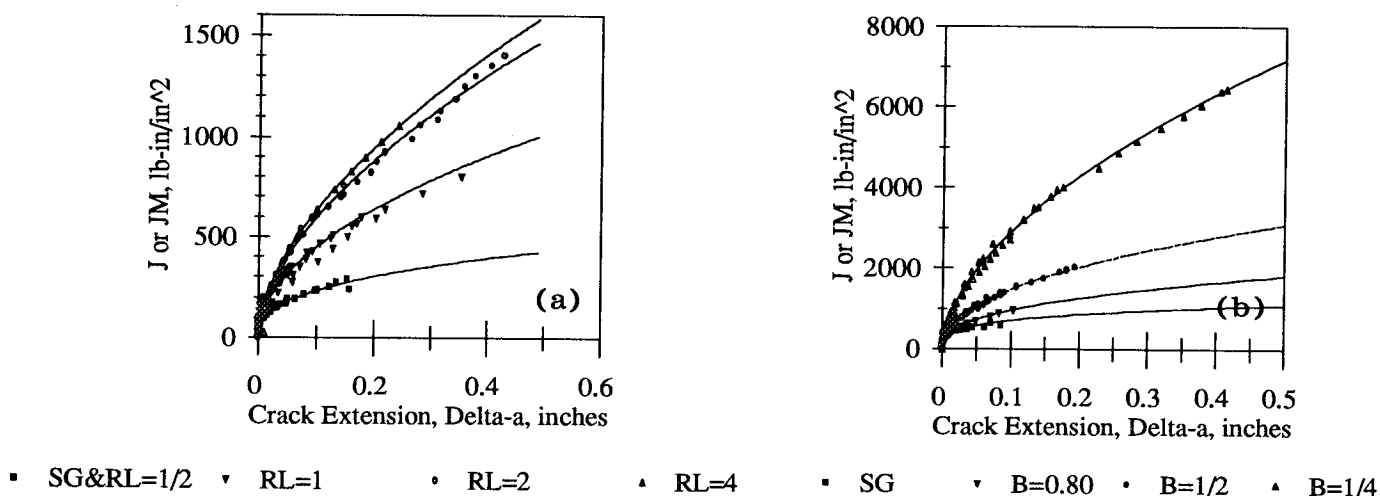


Figure 6: Results of application of inflection point of $J_M R$ curve as a validity limiting criterion, also showing results of the regression analysis of the qualified data (a) aluminum alloy, and (b) nickel alloy.

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